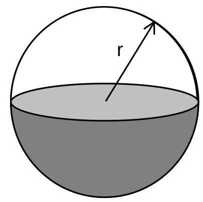
ÍNDICE

FORMULARIO DE MATEMÁTICAS

GEOMETRÍA

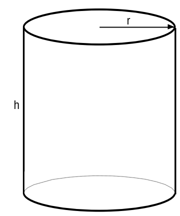
Volumen $= \frac{4}{3}\pi r^3$

Área de la Superficie$= 4\pi r^2$



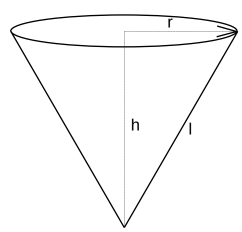
Volumen $= \pi r^2 h$

Área de la superficie lateral $= 2\pi rh$



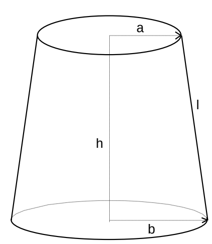
Volumen $= \frac{1}{3}\pi r^2 h$

Área de la superficie lateral $= \pi r\sqrt{r^2 + h^2} = \pi r\, l$



Volumen $= \frac{1}{3}\pi h\left(a^2 + ab + b^2\right)$

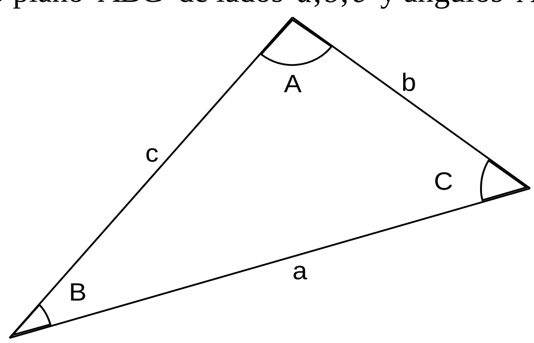
Área de la superficie lateral $\begin{aligned} &= \pi(a+b)\sqrt{h^2 + (b-a)^2} \\ &= \pi(a+b)\, l \end{aligned}$



TRIGONOMETRÍA

$\operatorname{sen}^2 A+\cos^2 A=1$	$\operatorname{sen}^2 A=\frac{1}{2}-\frac{1}{2}\cos 2A$
$\sec^2 A-\tan^2 A=1$	$\cos^2 A=\frac{1}{2}+\frac{1}{2}\cos 2A$
$\csc^2 A-\cot^2 A=1$	$\operatorname{sen}2A=2\operatorname{sen}A\cos A$
$\operatorname{sen}A\csc A=1$	$\cos 2A=\cos^2 A-\operatorname{sen}^2 A$
$\cos A\sec A=1$	$\operatorname{sen}(A\pm B)=\operatorname{sen}A\cos B\pm\cos A\operatorname{sen}B$
$\tan A\cot A=1$	$\cos(A\pm B)=\cos A\cos B\mp\operatorname{sen}A\operatorname{sen}B$
$\operatorname{sen}(-A)=-\operatorname{sen}A$	$tan(A\pm B)=\dfrac{tanA\pm tanB}{1\mp tanAtanB}$
$\cos(-A)=\cos A$	$\operatorname{sen}\dfrac{A}{2}=\pm\sqrt{\dfrac{1-\cos A}{2}}$
$\tan(-A)=-\tan A$	$\cos\dfrac{A}{2}=\pm\sqrt{\dfrac{1+\cos A}{2}}$
$\operatorname{sen}A\cos B=\frac{1}{2}\left[\operatorname{sen}(A-B)+\operatorname{sen}(A+B)\right]$	$\operatorname{sen}A\operatorname{sen}B=\frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]$
	$\cos A\cos B=\frac{1}{2}\left[\cos(A-B)+\cos(A+B)\right]$

Sea el siguiente triángulo plano ABC de lados a,b,c y ángulos A,B,C.



Ley de los senos	$\dfrac{a}{\operatorname{sen}A}=\dfrac{b}{\operatorname{sen}B}=\dfrac{c}{\operatorname{sen}C}$	
Ley de los cosenos	$c^2=a^2+b^2-2ab\cos C$	Los otros lados y ángulos están relacionados en forma similar
Ley de las tangentes	$\dfrac{a+b}{a-b}=\dfrac{tan\frac{1}{2}(A+B)}{tan\frac{1}{2}(A-B)}$	Los otros lados y ángulos están relacionados en forma similar

NÚMEROS COMPLEJOS

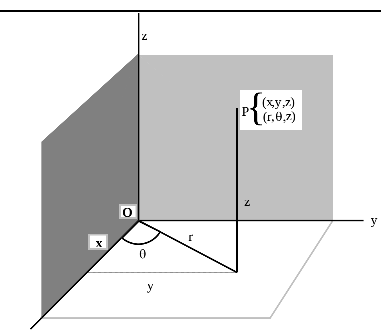
Teorema de DeMoivre	$\left[r(\cos\theta+i\operatorname{sen}\theta)\right]^n=r^n(\cos n\theta+i\operatorname{sen}n\theta)$	n: número entero
Raíz compleja	$\left[r(\cos\theta+isen\theta)\right]^{\frac{1}{n}}=r^{\frac{1}{n}}\left[\cos\left(\frac{\theta+2k\pi}{n}\right)+isen\left(\frac{\theta+2k\pi}{n}\right)\right]$	n: número entero positivo $k=0,1,2,\cdots,n-1$

GEOMETRÍA ANALÍTICA DEL ESPACIO

Considerando $P_1(x_1, y_1, z_1)$ y $P_2(x_2, y_2, z_2)$:

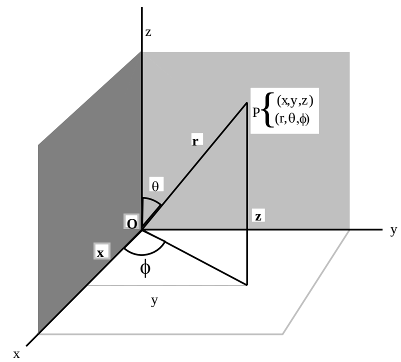
Vector que une P_1 y P_2	$\overline{P_1P_2} = \langle (x_2 - x_1), (y_2 - y_1), (z_2 - z_1) \rangle = \langle l, m, n \rangle$
Distancia entre dos puntos	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{l^2 + m^2 + n^2}$
Recta que pasa por dos puntos	Forma paramétrica $x = x_1 + lt \qquad y = y_1 + mt \qquad z = z_1 + nt$
	Forma simétrica $t = \frac{x - x_1}{l} \qquad t = \frac{y - y_1}{m} \qquad t = \frac{z - z_1}{n}$
Cosenos Directores	$\cos\alpha = \frac{x_2 - x_1}{d} = \frac{l}{d} \quad \cos\beta = \frac{y_2 - y_1}{d} = \frac{m}{d} \quad \cos\gamma = \frac{z_2 - z_1}{d} = \frac{n}{d}$
	donde α, β, γ ángulos que forman la línea que une los puntos P_1 y P_2 con la parte positiva de los ejes x, y, z, respectivamente
	$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \qquad l^2 + m^2 + n^2 = 1$
Ecuación del Plano	Que pasa por un punto $P_1(x_1, y_1, z_1)$ y tiene vector normal $\vec{n} = \langle n_1, n_2, n_3 \rangle$ $n_1(x - x_1) + n_2(y - y_1) + n_3(z - z_1) = 0$
	Forma general $Ax + By + Cz + D = 0$
	Distancia del punto $P_0(x_0, y_0, z_0)$ al plano $Ax + By + Cz + D = 0$ $d = \frac{\lvert Ax_0 + By_0 + Cz_0 + D \rvert}{\sqrt{A^2 + B^2 + C^2}}$
	Ángulo entre dos rectas en el plano $\tan\alpha = \frac{m_2 - m_1}{1 + m_1 m_2}$

Coordenadas:

Cilíndricas (r, θ, z) $\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \\ z = z \end{cases}$ o $\begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = tan^{-1}\left(\frac{y}{x}\right) \\ z = z \end{cases}$	

Esféricas (r,θ,ϕ)

$$\begin{cases} x = r\ \text{sen}\,\theta\ \cos\phi \\ y = r\ \text{sen}\,\theta\ \text{sen}\,\phi \\ z = r\ \cos\theta \end{cases} \quad \text{o} \quad \begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \phi = \tan^{-1}\left(\frac{y}{x}\right) \quad \text{con} \quad x \neq 0 \\ \theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) \end{cases}$$



REGLAS GENERALES DE DERIVACIÓN

$\frac{d}{dx}(c) = 0$	$\frac{d}{dx}(uvw) = u\,v\frac{dw}{dx} + u\,w\frac{dv}{dx} + v\,w\frac{du}{dx}$
$\frac{d}{dx}(cx) = c$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\left(\frac{du}{dx}\right) - u\left(\frac{du}{dx}\right)}{v^2}$
$\frac{d}{dx}\left(cx^n\right) = ncx^{n-1}$	$\frac{d}{dx}\left(u^n\right) = nu^{n-1}\frac{du}{dx}$
$\frac{d}{dx}\left(u \pm v\right) = \frac{du}{dx} \pm \frac{dv}{dx}$	$\frac{dF}{dx} = \frac{dF}{du}\frac{du}{dx}$ (Regla de la cadena)
$\frac{d}{dx}(cu) = c\frac{du}{dx}$	$\frac{du}{dx} = \frac{1}{\frac{dx}{du}}$
$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	$\frac{dF}{dx} = \frac{\frac{dF}{du}}{\frac{dx}{du}}$

Derivadas de las Funciones Exponenciales y Logarítmicas

$\frac{d}{dx}\log_a u = \frac{\log_a e}{u}\frac{du}{dx} \qquad a > 0, \quad a \neq 1$
$\frac{d}{dx}\ln u = \frac{d}{dx}\log_e u = \frac{1}{u}\frac{du}{dx}$
$\frac{d}{dx}a^u = a^u \ln a\frac{du}{dx}$
$\frac{d}{dx}e^u = e^u\frac{du}{dx}$
$\frac{d}{dx}u^v = \frac{d}{dx}e^{v\ln u} = e^{v\ln u}\frac{d}{dx}\left[v\ln u\right] = vu^{v-1}\frac{du}{dx} + u^v \ln u\frac{dv}{dx}$

Derivadas de las Funciones Trigonométricas y de las Trigonométricas Inversas

$\frac{d}{dx}\operatorname{sen} u = \cos u \frac{du}{dx}$	$\frac{d}{dx}\cos u = -\operatorname{sen} u \frac{du}{dx}$
$\frac{d}{dx}\tan u = \sec^2 u \frac{du}{dx}$	$\frac{d}{dx}\cot u = -\csc^2 u \frac{du}{dx}$
$\frac{d}{dx}\sec u = \sec u \tan u \frac{du}{dx}$	$\frac{d}{dx}\csc u = -\csc u \cot u \frac{du}{dx}$
$\frac{d}{dx}\operatorname{sen}^{-1} u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$ $\left[-\frac{\pi}{2} < \operatorname{sen}^{-1} u < \frac{\pi}{2}\right]$	$\frac{d}{dx}\cos^{-1} u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$ $\left[0 < \cos^{-1} u < \pi\right]$
$\frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx}$ $\left[-\frac{\pi}{2} < \tan^{-1} u < \frac{\pi}{2}\right]$	$\frac{d}{dx}\cot^{-1} u = \frac{-1}{1+u^2}\frac{du}{dx}$ $\left[0 < \cot^{-1} u < \pi\right]$
$\frac{d}{dx}\sec^{-1} u = \frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx} = \frac{\pm 1}{u\sqrt{u^2-1}}\frac{du}{dx}$ $\left[\begin{matrix} + & \text{si} & 0 < \sec^{-1} u < \frac{\pi}{2} \\ - & \text{si} & \frac{\pi}{2} < \sec^{-1} u < \pi \end{matrix}\right]$	
$\frac{d}{dx}\csc^{-1} u = \frac{-1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx} = \frac{\mp 1}{u\sqrt{u^2-1}}\frac{du}{dx}$ $\left[\begin{matrix} - & \text{si} & 0 < \csc^{-1} u < \frac{\pi}{2} \\ + & \text{si} & -\frac{\pi}{2} < \csc^{-1} u < 0 \end{matrix}\right]$	

Derivadas de las Funciones Hiperbólicas y de las Hiperbólicas Recíprocas

$\frac{d}{dx}\sinh u = \cosh u \frac{du}{dx}$	$\frac{d}{dx}\cosh u = \sinh u \frac{du}{dx}$
$\frac{d}{dx}\tanh u = \operatorname{sech}^2 u \frac{du}{dx}$	$\frac{d}{dx}\coth u = -\operatorname{csch}^2 u \frac{du}{dx}$
$\frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u \tanh u \frac{du}{dx}$	$\frac{d}{dx}\operatorname{csch} u = -\operatorname{csch} u \coth u \frac{du}{dx}$
$\frac{d}{dx}\operatorname{sen}\mathrm{h}^{-1} u = \frac{1}{\sqrt{u^2+1}}\frac{du}{dx}$	$\frac{d}{dx}\cos\mathrm{h}^{-1} u = \frac{\pm 1}{\sqrt{u^2-1}}\frac{du}{dx}$ $\left[\begin{matrix} + & \text{si} & \cosh^{-1} u > 0, u > 1 \\ - & \text{si} & \cosh^{-1} u < 0, u < 1 \end{matrix}\right]$
$\frac{d}{dx}\tanh^{-1} u = \frac{1}{1-u^2}\frac{du}{dx}$ $\left[-1 < u < 1\right]$	$\frac{d}{dx}\coth^{-1} u = \frac{1}{1-u^2}\frac{du}{dx}$ $\left[u > 1 \text{ o } u < -1\right]$
$\frac{d}{dx}\sec\mathrm{h}^{-1} u = \frac{\pm 1}{u\sqrt{u^2-1}}\frac{du}{dx}$ $\left[\begin{matrix} - & \text{si} & \operatorname{sech}^{-1} u > 0, & 0 < u < 1 \\ + & \text{si} & \sec\mathrm{h}^{-1} u < 0, & 0 < u < 1 \end{matrix}\right]$	
$\frac{d}{dx}\csc\mathrm{h}^{-1} u = \frac{-1}{\lvert u\rvert\sqrt{1+u^2}}\frac{du}{dx} = \frac{\mp 1}{u\sqrt{1+u^2}}\frac{du}{dx}$ $\left[\begin{matrix} - & \text{si} & u > 0 \\ + & \text{si} & u < 0 \end{matrix}\right]$	

TABLAS DE INTEGRALES

$$\int u\,dv = uv - \int v\,du$$

$$\int u^n du = \frac{1}{n+1} u^{n+1} + C \qquad n \neq -1$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int e^u\,du = e^u + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\int \operatorname{sen} u\,du = -\cos u + C$$

$$\int \cos u\,du = \operatorname{sen} u + C$$

$$\int \sec^2 u\,du = \tan u + C$$

$$\int \csc^2 u\,du = -\cot u + C$$

$$\int \sec u \tan u\,du = \sec u + C$$

$$\int \csc u \cot u\,du = -\csc u + C$$

$$\int \tan u\,du = \ln|\sec u| + C$$

$$\int \cot u\,du = \ln|\operatorname{sen} u| + C$$

$$\int \sec u\,du = \ln|\sec u + \tan u| + C$$

$$\int \csc u\,du = \ln|\csc u - \cot u| + C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$$

$$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$$

$$\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right| + C$$

$$\int u^2\sqrt{a^2+u^2}\,du = \frac{u}{8}(a^2+2u^2)\sqrt{a^2+u^2} - \frac{a^2}{8}\ln\left|u+\sqrt{a^2+u^2}\right| + C$$

$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\ln\left|\frac{\sqrt{a^2+u^2}+a}{u}\right| + C$$

$$\int \frac{du}{u^2\sqrt{a^2+u^2}} = -\frac{\sqrt{a^2+u^2}}{a^2u} + C$$

$$\int \frac{\sqrt{a^2+u^2}}{u}\,du = \sqrt{a^2+u^2} - a\ln\left|\frac{a+\sqrt{a^2+u^2}}{u}\right| + C$$

$$\int \frac{\sqrt{a^2+u^2}}{u^2}\,du = -\frac{\sqrt{a^2+u^2}}{u} + \ln\left|u+\sqrt{a^2+u^2}\right| + C$$

$$\int \frac{du}{\sqrt{a^2+u^2}} = \ln\left|u+\sqrt{a^2+u^2}\right| + C$$

$$\int \frac{u^2du}{\sqrt{a^2+u^2}} = \frac{u}{2}\sqrt{a^2+u^2} - \frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right| + C$$

$$\int \frac{du}{(a^2+u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2+u^2}} + C$$

$$\int \sqrt{a^2-u^2}\,du =$$

$$\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int u^2\sqrt{a^2-u^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{a^2-u^2} + \frac{a^4}{8}\operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int \frac{\sqrt{a^2-u^2}}{u}\,du = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$$

$$\int \frac{\sqrt{a^2-u^2}}{u^2}\,du = -\frac{1}{u}\sqrt{a^2-u^2} - \operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int \frac{u^2du}{\sqrt{a^2-u^2}} = -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$$

$$\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{1}{a^2u}\sqrt{a^2-u^2} + C$$

$$\int (a^2-u^2)^{3/2}du = -\frac{u}{8}(2u^2-5a^2)\sqrt{a^2-u^2} + \frac{3a^4}{8}\operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{(a^2-u^2)^{\frac{3}{2}}} = \frac{u}{a^2\sqrt{a^2-u^2}} + C$$

$$\int \frac{udu}{a+bu} = \frac{1}{b^2}(a+bu-a\ln|a+bu|) + C$$

$$\int \frac{u^2du}{a+bu} = \frac{1}{2b^3}\left[(a+bu)^2-4a(a+bu)+2a^2\ln|a+bu|\right] + C$$

$$\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$$

$$\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$$

$$\int u^2\sqrt{u^2-a^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{u^2-a^2} - \frac{a^4}{8}\ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int \frac{\sqrt{u^2-a^2}}{u}du = \sqrt{u^2-a^2} - a\cos^{-1}\frac{a}{u} + C$$

$$\int \frac{\sqrt{u^2-a^2}}{u^2}du = -\frac{\sqrt{u^2-a^2}}{u} + \ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int \frac{u^2du}{\sqrt{u^2-a^2}} = \frac{u}{2}\sqrt{u^2-a^2} + \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int \frac{du}{u^2\sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2u} + C$$

$$\int \frac{du}{(u^2-a^2)^{\frac{3}{2}}} = -\frac{u}{a^2\sqrt{u^2-a^2}} + C$$

$$\int \frac{u^2du}{\sqrt{a+bu}} = \frac{2}{15b^3}(8a^2+3b^2u^2-4abu)\sqrt{a+bu}$$

$$\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C, \text{ si } a>0$$

$$= \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bu}{-a}} + C, \text{ si } a<0$$

$$\int \frac{\sqrt{a+bu}}{u^2}du = -\frac{\sqrt{a+bu}}{u} + \frac{b}{2}\int \frac{du}{u\sqrt{a+bu}}$$

$$\int \frac{udu}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b}\ln|a+bu| + C$$

$$\int u^n\sqrt{a+bu}\,du = \frac{2}{b(2n+3)}\left[u^n(a+bu)^{3/2} - na\int u^{n-1}\sqrt{a+bu}\,du\right]$$

$$\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$$

$$\int \frac{u^2du}{(a+bu)^2} = \frac{1}{b^3}\left(a+bu-\frac{a^2}{a+bu}-2a\ln|a+bu|\right) + C$$

$$\int u\sqrt{a+bu}du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{\frac{3}{2}} + C$$

$$\int \frac{udu}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu}$$

$$\int \frac{u^ndu}{\sqrt{a+bu}} = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2na}{b(2n+1)}\int \frac{u^{n-1}du}{\sqrt{a+bu}}$$

$$\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int \frac{du}{u^{n-1}\sqrt{a+bu}}$$

$$\int \csc^3 u\,du = -\tfrac{1}{2}\csc u\cot u + \tfrac{1}{2}\ln|\csc u-\cot u| + C$$

$$\int \operatorname{sen}^n u\,du = -\tfrac{1}{n}\operatorname{sen}^{n-1}u\cos u + \frac{n-1}{n}\int \operatorname{sen}^{n-2}u\,du$$

$$\int \operatorname{sen}^2 u\,du = \tfrac{1}{2}u - \tfrac{1}{4}\operatorname{sen}2u + C$$

$$\int \cos^2 u\,du = \tfrac{1}{2}u + \tfrac{1}{4}\operatorname{sen}2u + C$$

$$\int \cos^n u\,du = \tfrac{1}{n}\cos^{n-1}u\operatorname{sen}u + \frac{n-1}{n}\int \cos^{n-2}u\,du$$

$$\int \tan^n u\,du = \frac{1}{n-1}\tan^{n-1}u - \int \tan^{n-2}u\,du$$

$$\int \tan^2 u\,du = \tan u - u + C$$

$$\int \cot^2 u\,du = -\cot u - u + C$$

$$\int \operatorname{sen}^3 u\,du = -\tfrac{1}{3}\left(2+\operatorname{sen}^2 u\right)\cos u + C$$

$$\int \cos^3 u\,du = \tfrac{1}{3}\left(2+\cos^2 u\right)\operatorname{sen} u + C$$

$$\int \tan^3 u\,du = \tfrac{1}{2}\tan^2 u + \ln\left|\cos u\right| + C$$

$$\int \cot^3 u\,du = -\tfrac{1}{2}\cot^2 u - \ln\left|\operatorname{sen} u\right| + C$$

$$\int \sec^3 u\,du = \tfrac{1}{2}\sec u\,tanu + \tfrac{1}{2}\ln\left|\sec u + tanu\right| + C$$

$$\int \operatorname{sen} au\,\cos bu\,du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$$

$$\int u\operatorname{sen} u\,du = \operatorname{sen} u - u\cos u + C$$

$$\int u\cos u\,du = \cos u + u\operatorname{sen} u + C$$

$$\int u^n \operatorname{sen} u\,du = u^n\cos u + n\int u^{n-1}\cos u\,du$$

$$\int \cot^n u\,du = \frac{-1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u\,du$$

$$\int \sec^n u\,du = \frac{1}{n-1}\,tanu\sec^{n-2} u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du$$

$$\int \csc^n u\,du = \frac{1}{n-1}\cot u\csc^{n-2} u + \frac{n-2}{n-1}\int \csc^{n-2} u\,du$$

$$\int \operatorname{sen} au\,\operatorname{sen} bu\,du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} - \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$$

$$\int \cos au\,\cos bu\,du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} + \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$$

$$\int u^n\cos u\,du = u^n\operatorname{sen} u - n\int u^{n-1}\operatorname{sen} u\,du$$

$$\int \operatorname{sen}^n u\cos^m u\,du$$

$$= -\frac{\operatorname{sen}^{n-1} u\cos^{m+1} u}{n+m} + \frac{n-1}{n+m}\int \operatorname{sen}^{n-2} u\cos^m u\,du$$

$$= -\frac{\operatorname{sen}^{n+1} u\cos^{m-1} u}{n+m} + \frac{m-1}{n+m}\int \operatorname{sen}^{n} u\cos^{m-2} u\,du$$

$$\int u\cos^{-1} u\,du = \frac{2u^2-1}{4}\cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$$

$$\int u\tan^{-1} u\,du = \frac{u^2+1}{2}\tan^{-1} u - \frac{u}{2} + C$$

$$\int \operatorname{sen}^{-1} u\,du = u\operatorname{sen}^{-1} u + \sqrt{1-u^2} + C$$

$$\int \cos^{-1} u\,du = u\cos^{-1} u - \sqrt{1-u^2} + C$$

$$\int \tan^{-1} u\,du = u\tan^{-1} u - \tfrac{1}{2}\ln\left(1+u^2\right) + C$$

$$\int u\operatorname{sen}^{-1} u\,du = \frac{2u^2-1}{4}\operatorname{sen}^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$$

$$\int ue^{au}\,du = \frac{1}{a^2}(au-1)e^{au} + C$$

$$\int u^n e^{au}\,du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1}e^{au}\,du$$

$$\int e^{au}\operatorname{sen} bu\,du = \frac{e^{au}}{a^2+b^2}(a\operatorname{sen} bu - b\cos bu) + C$$

$$\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\operatorname{sen} bu) + C$$

$$\int u^n\operatorname{sen}^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\operatorname{sen}^{-1} u - \int \frac{u^{n+1}du}{\sqrt{1-u^2}}\right],\quad n\neq -1$$

$$\int u^n\cos^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\cos^{-1} u + \int \frac{u^{n+1}du}{\sqrt{1-u^2}}\right],\quad n\neq -1$$

$$\int u^n\tan^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\tan^{-1} u - \int \frac{u^{n+1}du}{\sqrt{1+u^2}}\right], n\neq -1$$

$$\int \ln u\,du = u\ln u - u + C$$

$$\int u^n\ln u\,du = \frac{u^{n+1}}{(n+1)^2}\left[(n+1)\ln u - 1\right] + C$$

$$\int \frac{1}{u\ln u}\,du = \ln\left|\ln u\right| + C$$

$$\int \operatorname{senh} u\, du = \cosh u + C$$

$$\int \cosh u\, du = \operatorname{senh} u + C$$

$$\int \tanh u\, du = \ln \cosh u + C$$

$$\int \coth u\, du = \ln\left|\operatorname{senh} u\right| + C$$

$$\int \operatorname{sech} u\, du = tan^{-1}\left|\operatorname{senh} u\right| + C$$

$$\int \operatorname{sech} u\, du = \ln\left|\tan \tfrac{1}{2} u\right| + C$$

$$\int \operatorname{sech}^2 u\, du = \tanh u + C$$

$$\int \operatorname{csch}^2 u\, du = -\coth u + C$$

$$\int \operatorname{sech} u \tanh u\, du = -\operatorname{sech} u + C$$

$$\int \operatorname{csch} u \coth u\, du = -\operatorname{csch} u + C$$

$$\int \sqrt{2au-u^2}\, du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$\int u\sqrt{2au-u^2}\, du = \frac{2u-au-3a^2}{6}\sqrt{2au-u^2} + \frac{a^3}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$\int \frac{\sqrt{2au-u^2}}{u^2}\, du = \sqrt{2au-u^2} + a\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$\int \frac{\sqrt{2au-u^2}}{u^2}\, du = -\frac{2\sqrt{2au-u^2}}{u} - \cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$\int \frac{u^2 du}{\sqrt{2au-u^2}} = -\frac{(u+3a)}{2}\sqrt{2au-u^2} + \frac{3a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$\int \frac{du}{\sqrt{2au-u^2}} = \cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$\int \frac{u\, du}{\sqrt{2au-u^2}} = -\sqrt{2au-u^2} + a\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$\int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au} + C$$

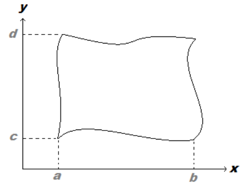
VECTORES

Producto punto	$\lvert \mathbf{A}\cdot\mathbf{B}\rvert = \lVert A\rVert \lVert B\rVert \cos\theta \qquad 0\le\theta\le\pi$ donde θ es el ángulo formado por **A** y **B**	
	$\mathbf{A}\cdot\mathbf{B} = A_1B_1 + A_2B_2 + A_3B_3$ donde $\mathbf{A} = \langle A_1, A_2, \cdots A_n\rangle$ y $\mathbf{B} = \langle B_1, B_2, \cdots B_n\rangle$	
Producto cruz	$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \hat{\text{i}} & \hat{\text{j}} & \hat{\text{k}} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$ $= (A_2B_3 - A_3B_2)\hat{\mathbf{i}} + (A_3B_1 - A_1B_3)\hat{\mathbf{j}} + (A_1B_2 - A_2B_1)\hat{\mathbf{k}}$ donde $\mathbf{A} = A_1\hat{\text{i}} + A_2\hat{\text{j}} + A_3\hat{\text{k}}$ y $\text{B} = B_1\hat{\text{i}} + B_2\hat{\text{j}} + B_3\hat{\text{k}}$	
	Magnitud del producto cruz $\lVert\mathbf{A}\times\mathbf{B}\rVert = \lVert\mathbf{A}\rVert\lVert\mathbf{B}\rVert \operatorname{sen}\theta$	

Sean $U = U(x, y, z)$, una función escalar, y $\mathbf{A} = \mathbf{A}(x, y, z)$, una función vectorial,ambas con derivadas parciales

Operador *nabla*	$\nabla = \frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}$
Gradiente de U	$grad\ U = \nabla U = \left(\frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}\right)U = \frac{\partial U}{\partial x}\hat{\mathbf{i}} + \frac{\partial U}{\partial y}\hat{\mathbf{j}} + \frac{\partial U}{\partial z}\hat{\mathbf{k}}$
Laplaciano de U	$\nabla^2 U = \nabla\cdot(\nabla U) = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$
Divergencia de **A**	$div\ \mathbf{A} = \nabla\cdot\mathbf{A} = \left(\frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}\right)\cdot\left(A_1\hat{\mathbf{i}} + A_2\hat{\mathbf{j}} + A_3\hat{\mathbf{k}}\right)$ $= \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$
Rotacional de **A**	$rot\ \mathbf{A} = \nabla\times\mathbf{A} = \left(\frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}\right)\times\left(A_1\hat{\mathbf{i}} + A_2\hat{\mathbf{j}} + A_3\hat{\mathbf{k}}\right)$ $= \begin{vmatrix} \hat{\text{i}} & \hat{\text{j}} & \hat{\text{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$ $= \left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right)\hat{\text{i}} + \left(\frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial x}\right)\hat{\text{j}} + \left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right)\hat{\text{k}}$

INTEGRALES MÚLTIPLES

Integrales dobles o integrales de área 	$\int_{x=a}^{b}\int_{y=f_1(x)}^{f_2(x)} F(x,y)\,dydx = \int_{x=a}^{b}\left\{\int_{y=f_1(x)}^{f_2(x)} F(x,y)\,dy\right\}dx$ $\int_{y=c}^{d}\int_{x=g_1(y)}^{g_2(y)} F(x,y)\,dxdy = \int_{y=c}^{d}\left\{\int_{x=g_1(y)}^{g_2(y)} F(x,y)\,dx\right\}dy$
Los anteriores conceptos se pueden ampliar para considerar integrales triples o de volumen así como integrales múltiples en más de tres dimensiones.	

	En parámetro arbitrario:	En parámetro s:
Vector tangente unitario	$\hat{t}(t) = \dfrac{\vec{r}'(t)}{\lVert\vec{r}'(t)\rVert}$	$\hat{t}(s) = \dot{\vec{r}}(s)$
Vector normal principal	$\hat{n}(t) = \hat{b}(t)\times\hat{t}(t)$	$\hat{n}(s) = \dfrac{\ddot{\vec{r}}(s)}{\lVert\ddot{\vec{r}}(s)\rVert}$
Vector binormal	$\hat{b}(t) = \dfrac{\vec{r}'\times\vec{r}''(t)}{\lVert\vec{r}'\times\vec{r}''(t)\rVert}$	$\hat{b}(s) = \dfrac{\dot{\vec{r}}(s)\times\ddot{\vec{r}}(s)}{\lVert\ddot{\vec{r}}(s)\rVert}$
Los vectores unitarios $\hat{t},\hat{n},\hat{b}$ guardan la relación $\hat{b} = \hat{t}\times\hat{n},\ \hat{n} = \hat{b}\times\hat{t},\ \hat{t} = \hat{n}\times\hat{b}$		

Recta tangente en t_0	Ecuación vectorial $\vec{r}(\lambda) = \vec{r}(t_0) + \lambda\vec{r}\,'(t_0)$
	Ecuación paramétrica $\dfrac{x-x_0}{x_0'} = \dfrac{y-y_0}{y_0'} = \dfrac{z-z_0}{x_0'}$
Plano osculador$(\hat{t},\hat{n})$ en t_0	Ecuación vectorial $\left[\vec{r}-\vec{r}(t_0)\right]\cdot\left[\vec{r}'(t_0)\times\vec{r}''(t_0)\right] = 0$
	Ecuación paramétrica $\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ x_0' & y_0' & z_0' \\ x_0'' & y_0'' & z_0'' \end{vmatrix} = 0$
Plano normal	Ecuación vectorial $\left(\vec{r}-\vec{r}(t_0)\right)\cdot\vec{r}\,'(t_0) = 0$
	Ecuación paramétrica $x_0'(x-x_0) + y_0'(y-y_0) + z_0'(z-z_0) = 0$
Plano Rectificante$(\hat{t},\hat{b})$ en t_0	Ecuación vectorial $\left(\vec{r}-\vec{r}(t_0)\right)\cdot\hat{n}(t_0) = 0$
	Ecuación paramétrica $\begin{vmatrix} x - x_0 & y - y_0 & z - z_0 \\ x_0' & y_0' & z_0' \\ y_0'z_0'' - y_0''z_0' & z_0'x_0'' - z_0''x_0' & x_0'y_0'' - x_0''y_0' \end{vmatrix} = 0$

Curvatura y Torsión	$\kappa(t)=\frac{\left\|\vec{r}'(t)\times\vec{r}''(t)\right\|}{\left\|\vec{r}'(t)\right\|^3}$ $\quad\tau(t)=\frac{\vec{r}'(t)\cdot\left[\vec{r}''(t)\times\vec{r}'''(t)\right]}{\left\|\vec{r}'(t)\times\vec{r}''(t)\right\|^2}$ $\kappa(s)=\left\|\ddot{\vec{r}}(s)\right\|$ $\quad\kappa=\frac{\lvert f''(x)\rvert}{[1+(f'(x))^2]^{3/2}}$
Componentes Tangencial de la Aceleración	$a_T=\vec{a}\cdot\vec{T}=\frac{\vec{v}\cdot\vec{a}}{\left\|\vec{v}\right\|}$
Componentes Normal de la Aceleración	$a_N=\vec{a}\cdot\vec{N}=\frac{\left\|\vec{v}\times\vec{a}\right\|}{\left\|\vec{v}\right\|}$
Propiedades de la Divergencia	$\nabla\cdot\left(\vec{F}+\vec{G}\right)=\nabla\cdot\vec{F}+\nabla\cdot\vec{G}$
	$\nabla\cdot\left(\phi\vec{F}\right)=\phi\nabla\cdot\vec{F}+\left(\nabla\phi\right)\cdot\vec{F}$
	$\nabla\cdot\left(\vec{F}+\vec{G}\right)=\vec{G}\cdot\left(\nabla\times\vec{F}\right)-\vec{F}\cdot\left(\nabla\times\vec{G}\right)$

TRANSFORMADA DE LAPLACE

$$\mathrm{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\, dt$$

No	**f(t)**	**F(s)**
1	C (*constante*)	$\frac{C}{s}$
2	t^n	$\frac{n!}{s^{n+1}}$, $n = 0$ y $n \in \mathrm{N}$
3	t^n	$\frac{\Gamma(n+1)}{s^{n+1}}$, $n > -1$
4	e^{at}	$\frac{1}{s-a}$
5	$senh(at)$	$\frac{a}{s^2 - a^2}$
6	$cosh(at)$	$\frac{s}{s^2 - a^2}$
7	$sen(kt)$	$\frac{k}{s^2 + k^2}$
8	$\cos(kt)$	$\frac{s}{s^2 + k^2}$
9	$e^{at} f(t)$	$F(s-a)$
10	$f(t-a)U(t-a)$	$e^{-as} F(s)$
11	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
12	$\frac{f(t)}{t}$	$\int_s^{\infty} F(p)\,dp$
13	$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots\ f^{(n-1)}(0)$
14	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
15	$f * g = \int_0^t f(\tau) g(t-\tau)\,d\tau$	$F(s)G(s)$
16	$f(t)$ función periódica de periodo T	$\frac{1}{1-e^{-sT}} \int_0^T f(t) e^{-st}\,dt$
17	$\delta(t)$	1
18	$\delta(t - t_0)$	$e^{-t_0 s}$

FÓRMULAS MISCELÁNEAS

Área en coordenadas polares	$\frac{1}{2}\int_{\alpha}^{\beta} r^2 dr$
Ecuaciones paramétricas de la cicloide para $t \in R$	$x = a(t - \operatorname{sen} t)$ $y = a(1 - \cos t)$
Trabajo	$W = \int_a^b \vec{F} \cdot \overrightarrow{dr}$ $\quad Comp(\vec{a}_{\vec{b}}) = \frac{\lvert \vec{a} \cdot \vec{b} \rvert}{\lVert \vec{b} \rVert}$
Longitud de arco de $y = f(x)$ en $[a,b] = \int_a^b \sqrt{1+(y')^2}\,dx$	$m = \iint_R \rho(x,y)dA$ $M_x = \iint_R y\rho(x,y)\,dA$ $\quad M_y = \iint_R x\rho(x,y)\,dA$
Centro de gravedad de una región plana	$\overline{x} = \frac{\int_a^b xf(x)dx}{\int_a^b f(x)dx}$ $\quad \overline{y} = \frac{\frac{1}{2}\int_a^b [f(x)]^2\,dx}{\int_a^b f(x)dx}$
Longitud de arco en forma paramétrica	$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
Momento de inercia de R respecto al origen	$I_o = \iint_R (x^2 + y^2)\rho(x,y)\,dA$
Área de la superficie generada al girar la gráfica f alrededor de x	$S = \int_a^b 2\pi F(x)\sqrt{1+(f(x))^2}\,dx$
Volumen del sólido de revolución generado al girar la gráfica de f alrededor del eje y	$V = \int_a^b 2\ \pi t F(t)dt$
Cálculo del volumen	$V = \int_a^b A(x)\,dx$ $\quad V = \int_a^b \pi(f(x))^2\,dx$
Ecuación del resorte helicoidal	$\vec{r}(t) = \left(\cos t, \operatorname{sen} t, \frac{t}{2\pi}\right)$
Derivada direccional	$D_{\hat{u}} f(x,y,z) = \nabla f(x,y,z) \cdot \hat{u}$ $\quad \hat{u}$: Vector unitario
Ecuación satisfecha por la carga de un circuito LRC	$Lq'' + Rq' + \frac{1}{C}q = E(t)$
Fuerza ejercida por un fluído	$F = \int_a^b \gamma\, y \cdot L(y)\,dy$
Fuerza que actúa sobre un líquido encerrado en un tubo	$F = \delta A 2 x_0 g - \delta A 2 x g$